

Assignment: Psychological perspectives your example assessment.  
The NAEP 8<sup>th</sup> grade Mathematics Assessment

The goal of NAEP mathematics is to demonstrate through its framework, the structure and specifications for the assessment items, to gather information about what students should know and be able to do in mathematics and identify appropriate achievement level at each age and grade. The framework (below), from *Mathematics framework for the 1996 and 2000 National Assessment of Educational Program*, shows the structure of this assessment.

### Mathematical Framework for the 1996 and 2000 Assessments

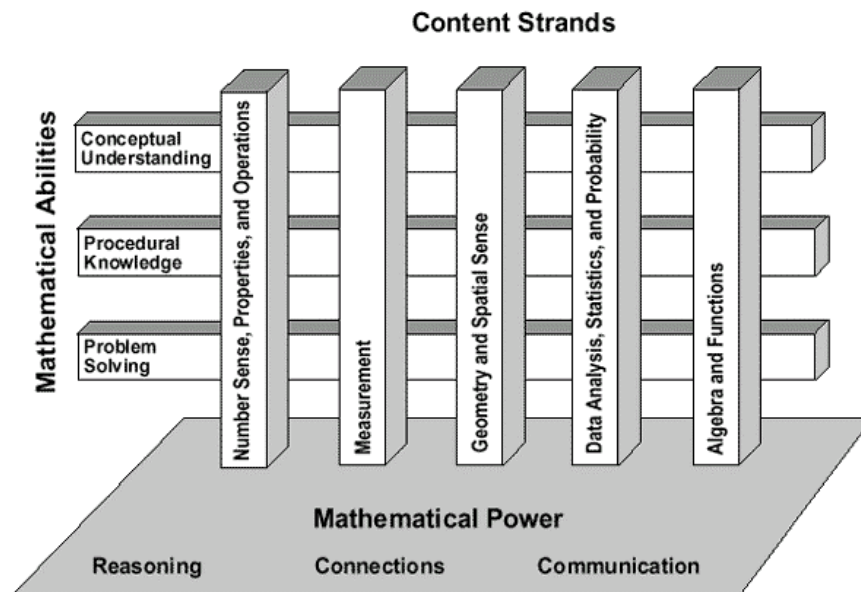


Figure 2. *Mathematics Framework for the 1996 and 2000 National Assessment of Educational Progress.*

As Messick pointed out, in designing an assessment we started with 1) What we want to make inference about, and 2) what we need to see to ground those inferences?

The assessment is carried out through three-dimensional scope contexts: Content Strands, Mathematical Abilities and Mathematical Power. The assessment items with particular attention to a mix of formats are designed to measure not only students' knowledge in the content strands, but also their mathematical abilities and mathematical power within a broad scope context of reasoning and with connections across broad mathematical content and thinking. Specifically, as following:

- I. Five mathematics content areas measure students' basic mathematical knowledge in each strands:
  - Number sense, properties, and operations
  - Measurement
  - Geometry and spatial sense
  - Data analysis, statistics, and probability
  - Algebra and functions
- II. Three types of mathematical abilities measure students' basic mathematical abilities:
  - Conceptual understanding

- Procedural knowledge
- Problem solving

III. Mathematical power extend the measures for students' mathematical abilities to their abilities to *reason* in mathematical situations, to *connect* knowledge in one mathematical area to other mathematical areas, and to *communicate* the perceptions and conclusions drawn from the mathematical situations:

- Reasoning
- Connections
- Communication

From the perspective of trait psychology, the target of inference is traits that presumably influenced performance over a wide range of circumstances and samples of those circumstances are needed (Mislevy, 2000). Five broad strands of mathematics contents follow this perspective. From the perspective of cognitive psychology, the target inference are cast in terms of patterns, skills and knowledge structures that characterize students' developing proficiency. Mathematical ability and mathematical power are intended to gather information and assess students from this perspective.

Messick quotes, a construct-centered approach would begin by asking what complex of knowledge and skills should be assessed, then what behaviors/performance should reveal those, and what tasks should elicit those behaviors? These define the basic three models: student model, evidence model and task model.

Student-model variables describe characteristics of examinees, they are derived from cognitive task analysis, instructional goals, and instructional approach and simulator capabilities (Mislevy, 199x). They consist of the five content strands, three types of mathematical abilities and mathematical power in the framework.

Evidence-model variables describe features of specific task performances. Evidence-rules extract the salient features from students' work product and use them to evaluate values of observable variables. They consist of response choices, drawings and explanations, scoring rules, and the statistical model to the assessment items.

Task-model variables describe features of tasks (specifications for the stimulus, work product). They consist of three types of items: multiple-choice items, open-ended items, and extended open-ended items. The new assessment items focus even more attention on mathematical power by continuing deliberate attention to reasoning and communication and by providing students with opportunities to connect their learning across mathematical content strands.

Now, I will discuss in each of the areas based on *Mathematics framework for the 1996 and 2000 National Assessment of Educational Program*, how these cognitive perspectives connect to the assessment and the basic three models and their components. Specifically, what we need to see to give us evidence about what we want to know using effective ways in each of the five content strands, and mathematical abilities and mathematical power.

## **Task Models**

Task models contain multiple-choice items, open-ended items and extended open-ended items. Multiple-choice items require students to read, reflect, or compute and then to select the options they believe to be the correct answer. Open-ended items are short-answer items. They require students to give either a numerical result or the correct name or classification for a group of mathematical objects, provide a drawing example of a given concept, or perhaps give a brief explanation. Extended open-ended items require students to provide information from one or more of the content strands and how they understand and solve them, and reason and connect to their knowledge. All three types of items contain specifications for the stimulus material for students to produce their answers as their work products. Figure 2, 3, 4 show an example of a multiple-choice item, a problem solving item and a reasoning item, as well as their specifications.

### Content Strands: Number Sense, Properties, and Operations

#### **Student Model**

This strand focuses on students' understanding of numbers (whole numbers, fractions, decimals, integers, real numbers, complex numbers, especially positive and negative numbers), properties and operations involving those numbers, estimations and use of ratios and proportional thinking to represent situations involving quantity and their application to real world situations. It also is interested in students' knowledge and understanding of scientific notation to represent large and small numbers, of relative size, equivalent forms of numbers, and their use of numbers to represent attributes of real-world objects and quantities.

#### **Evidence Model**

Students are expected to demonstrate an understanding of numerical relationships such as ratios, proportions, percentages, and properties of numbers and operations, generalization of numerical patterns, verification of results in the student model. Students are also expected to demonstrate their knowledge of performing basic algorithms and using calculators in appropriate ways in more complex situations.

### Content Strands: Measurement

#### **Student Model**

This strand focuses on students' understanding of the process of measurement and on their use of numbers and measures to describe and compare mathematical and real-world objects, also on students' understanding and usage of the measurement attributes of length, mass/weight, capacity, time, money, and temperature, as well as their abilities using volume or surface area to combine shapes, translate, and apply measures.

#### **Evidence Model**

Students are asked to identify attributes, select appropriate units and tools, apply measurement concepts, and communicate measurement-related ideas. They are expected to demonstrate their ability to extend basic concepts in application involving perimeter, area, surface area, volume, and angle measurement. They are also expected to solve problems involving proportional thinking and apply the use of complex measurement formulas, and illustrate the connections among number sense and operations, algebra, and geometry.

#### **Task Model**

In addition to the standard types of items, students can use measuring instruments and apply measurement concepts to solve problems.

### Content Strands: Geometry and Spatial Sense

#### **Student Model**

This strand focus on students' understanding of properties of angles and polygons, and their reasoning skills to make and validate conjectures about transformations and combinations of shapes, as well as their skills to extend and connect proportional thinking to similar figures and indirect measurement.

**Evidence Model**

Students are expected to provide informal construction and demonstrations including drawing representations, along with their justifications in addition to more traditional types of compass-and-straightedge constructions and proofs. They are expected to demonstrate their reasoning within both formal and informal settings and connecting proportional thinking to similar figures and indirect measurement.

**Content Strands: Data Analysis, Statistics, and Probability****Student Model**

This strand's focus on students' important skills of collecting, organizing, reading, representing, and interpreting data in a variety of contexts to reflect the pervasive use of these skills in dealing with information. It is interested in the students' understanding and application of statistics and statistical concepts beyond these basic skills which include analyzing and communicating increasingly sophisticated interpretations of data, dealing with uncertainty and making predictions about outcomes in problem-solving and decision-making situations.

**Evidence Model**

Students are expected to analyze statistical claims, design experiments and making decisions. They may use simulations to model real-world situations. They should demonstrate some understanding of sampling and make predictions based on experiments or data. They are expected to be comfortable with variety of graphs to represent different types of data in different situations.

**Task Model**

In addition to standard items, the questions in this strand emphasize the features of the strands including the appropriate methods of gathering data, the visual exploration of data, the variety of ways to represent data, and the development and evaluation of arguments based on data analysis.

**Content Strands: Algebra and Functions****Student Model**

This strand's focus on students' knowledge of algebra and basic concepts of functions, their understanding of equations as a modeling tool, and their skills to use variables, describe relationships and solve simple equations and inequalities by a variety of methods.

**Evidence Model**

Students are expected to demonstrate their skills in using algebraic notation and thinking in meaningful context, and using the notion of equivalent representations to transform and solve number sentences and equations for mathematical and real-world problems. They are expected, specifically, to demonstrate how to use functions including algebraic and geometric functions, how to use open sentences and equations as representational tools.

**Mathematical Abilities: Conceptual Understanding****Student Model**

Students' conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

**Evidence Model**

Students provide evidence on their conceptual understanding in mathematics when they: recognize, label, and generate examples and non-examples of concepts; use and interrelate models, diagrams, manipulatives, and varied representations of concepts; identify and apply principles; know and apply facts and definitions; compare, contrast, and integrate related concepts and principles to extend the nature of concepts and principles; recognize, interpret, and apply the signs, symbols, and terms used to represent concepts; or interpret the assumptions and relations involving concepts in mathematical settings. Students' abilities are reflected by their performance that indicates the production of examples, common or unique representations, or communication

indicating the ability to manipulate the central ideas about the understanding of a concept in a variety of ways.

### Mathematical Abilities: Procedural Knowledge

#### **Student Model**

Students' procedural knowledge includes the various numerical algorithms in mathematics and encompasses the ability to read and produce graphs and tables, execute geometric constructions, and perform non-computational skills such as rounding and ordering. It is often reflected in a student's ability to connect an algorithmic process with a given problem situation, to employ that algorithm correctly, and to communicate the results of the algorithm in the context of the problem setting.

#### **Evidence Model**

Students demonstrate procedural knowledge when they: select and apply appropriate procedures correctly; verify or justify the correctness of a procedure using concrete models or symbolic methods; or extend or modify procedures to deal with factors inherent in problem settings; also how well they execute a procedure or how well they select the appropriate procedure for a given task.

### Mathematical Abilities: Problem Solving

#### **Student Model**

Students' problem solving skill includes their ability to recognize and formulate problems; determine the sufficiency and consistency of data; use strategies, data, models, and relevant mathematics; generate, extend, and modify procedures; use reasoning such as spatial, inductive, deductive, statistical, or proportional reasoning in new settings; and judge the reasonableness and correctness of solutions.

#### **Evidence Model**

Students are required to use their accumulated knowledge of mathematics in new situations, to connect all of their mathematical knowledge of concepts, procedures, reasoning, and communication/representational skills in confronting new situations. And these situations are the most accurate measures of students' proficiency in mathematics.

### Mathematical power (Reasoning, Connections and Communication)

#### **Student Model**

Students' mathematical power is characterized as a overall ability to gather and use mathematical knowledge through exploring, conjecturing, and reasoning logically; through solving non-routine problems; through communicating about and through mathematics; and through connecting mathematical ideas in one context with mathematical ideas in another context or with ideas from another discipline in the same or related contexts. It is a function of their prior knowledge and experiences and the ability to connect that knowledge in productive ways to new contexts. Students' mental skills of reasoning, communicating and connecting lie at the foundation of each of the content strands and each of the mathematical abilities.

#### **Evidence Model**

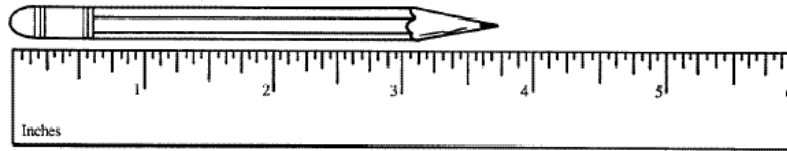
Students display their mathematical power through the formulations of lines of attack on problems and the way in which they reason through situations involving a multitude of possibilities. Through a student's report of his/her thinking, the questions of relevance of approach, nature of reasoning, and ability to solve problems becomes less a high inference guess and more of a conclusion that can be drawn from evidence.

#### **Task Model**

The hand calculators are allowed to help students to check alternative paths, check their directions or confirm their judgment made through other approaches.

**Appendix: Examples of Item Types**

Figure 1 is a sample of multiple-choice item from Measurement content strand. It is not only used to provide evidence about the understanding of measurement but also used to give evidence about students' procedural knowledge when they apply appropriate procedures to determine the correct length of the pencil.



3. What is the length of this pencil to the nearest quarter inch?

- A)  $3\frac{1}{4}$  inches
- B)  $3\frac{3}{4}$  inches
- C)  $4\frac{1}{4}$  inches
- D) 4 inches
- E) I don't know.

Figure 2 is a sample of multiple-choice item from Data analysis, statistics, and probability content strand. It is used to measure students' skill of collecting, organizing, sampling and interpreting data. It also allows students' demonstration of their problem solving ability, and their abilities to reason, connect and communicate the possibilities of the choices.

4. A bag contains two red candies and one yellow candy. Kim takes out one candy and eats it, and then Jeff takes out one candy. For each sentence below, fill in the oval to indicate whether it is possible or not possible.

Possible    Not Possible

- |                       |                       |   |
|-----------------------|-----------------------|---|
| <input type="radio"/> | <input type="radio"/> | Kim's candy is red and Jeff's candy is red.       |
| <input type="radio"/> | <input type="radio"/> | Kim's candy is red and Jeff's candy is yellow.    |
| <input type="radio"/> | <input type="radio"/> | Kim's candy is yellow and Jeff's candy is red.    |
| <input type="radio"/> | <input type="radio"/> | Kim's candy is yellow and Jeff's candy is yellow. |

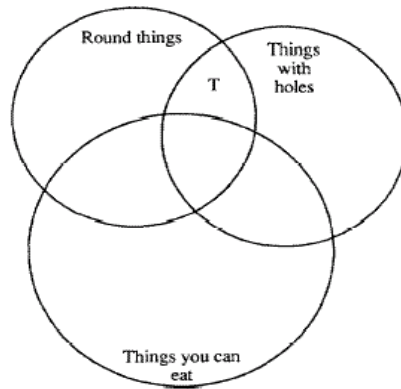
Figure 3 is a sample of open-ended item from Geometry and spatial sense content strand. It measures not only geometry sense but also problem solving ability. Through the process of identification, transformation and combination of the shapes, students will demonstrate their construction, proportional thinking, and use of strategy to connect all their knowledge, and reason and communicate them in the graph.

Each circle in the figure below represents the set of objects that have a certain property, as described in the circle. Where an object is placed in the figure depends on which of the properties it has.

Example: Note that the T (for tire) is placed in two of the circles at the same time, but not in the third. That is, it is in the circle that represents round things and the circle that represents things with holes, but it is not in the circle that represents things you can eat.



T - Tire



6. Indicate where each of the following objects should be placed in the diagram by writing its letter in the correct place. Use each letter only once.

		
M - Marble	D - Doughnut	S - Slice of Swiss Cheese