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An Illustration of Second-Order Latent Growth Models

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Methods of latent curve analysis (latent growth modeling) have recently emerged as a versatile tool for investigating longitudinal change in measured variables. This article, using higher order factor models as suggested by McArdle (1988) and Tisak and Meredith (1990), illustrates latent curve analysis for the purpose of modeling longitudinal change directly in a latent construct. The construct of interest is assumed to be indicated by several measured variables, all of which are observed at the same multiple time points. Examples with simultaneous estimation of covariance and mean structures are provided for both a single group and a two-group scenario.

The methods used to analyze longitudinal data are varied. These include univariate and multivariate analysis of variance, univariate and multivariate analysis of covariance, as well as auto-regressive and cross-lagged multiple regression techniques. Such techniques, if their underlying assumptions are met, are capable of providing the researcher with useful information about the behavior of groups as a whole. In their most common forms, however, these methods are not utilized to elucidate growth at the individual level.

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To help overcome some of the limitations of more traditional analytic approaches to the assessment of change over time, a class of methods has emerged fairly recently from the area of structural equation modeling. Such methods, falling under the general heading of latent curve analysis (LCA) or latent growth modeling, approach the analysis of growth from a somewhat different perspective than the aforementioned traditional methods. Specifically, LCA techniques may describe individuals’ behavior in terms of initial levels (or levels at other temporal reference points) and their developmental trajectories from and to those levels. In addition, they determine the variability across individuals in both initial levels and trajectories, as well as provide a means for testing the contribution of other variables or constructs to explaining those initial levels and growth trajectories. In doing so, LCA methods simultaneously focus on correlations over time, changes in variance, and shifts in mean values, thus utilizing more information available in the measured variables than do traditional methods. A very brief summary of such methods follows to preface the second-order procedures illustrated herein; readers desiring more background information are referred to a number of readable pieces on this topic (e.g., Duncan & Duncan, 1995; Duncan, Duncan, & Stoolmiller, 1994; Lawrence & Hancock, 1998; McArdle, 1988; McArdle & Epstein, 1987; Meredith, 1991; Meredith & Tisak, 1990; McArdle & Epstein, 1987; Meredith & Tisak, 1990; Muthén, 1991; Meredith & Tisak, 1990; Tisak & Meredith, 1990), including a recent book by Duncan, Duncan, Strycker, Li, and Alpert (1999).

**DEVELOPMENT**

Briefly, a model investigating longitudinal growth in a variable \( X \) could be expressed as \( x = \Lambda \xi + \delta \). The vector \( x \) contains values of \( X \) across time, \( \Lambda \) is a matrix of loadings reflecting the hypothesized growth pattern underlying \( X \), \( \xi \) is a vector of factors capturing the facets of growth being modeled, and \( \delta \) is a vector of random normal errors. A common example for linear growth would posit \( \xi = [\alpha \beta]' \), where \( \alpha \) is an intercept factor representing the true initial amount of \( X \) and \( \beta \) is a slope factor representing the true rate of change over time. Assuming (for simplicity) that the \( X \) variable is measured at four equal-interval time points, a test for linear growth could be conducted by fitting the following loadings:

\[
\Lambda = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix}
\]

This particular pattern makes \( \alpha \) represent the true initial amount of \( X \) and \( \beta \) represent the true rate of linear change over time from that initial amount \( \alpha \). More generally, for change in \( X \) measured across \( t \) time points, \( \xi \) may contain up to \( t - 1 \)
factors capturing polynomial growth up through the \((t-1)\)th functional order. For the interested reader, the first- and second-order moments for \(X\) are presented in Appendix A.

Models such as that described employ a first-order factor structure to investigate change in a measured variable \(X\) across time; for this reason they are referred to herein as first-order latent growth models. These first-order latent growth models have been used to analyze change in alcohol, cigarette, and marijuana use (Duncan & Duncan, 1994), tolerance of deviant behavior (Willett & Sayer, 1994), client resistance during parent training therapy (Stoolmiller, Duncan, Bank, & Patterson, 1993), family functioning (Willett, Ayoub, & Robinson, 1991), and antisocial behavior (Patterson, 1993).

This last example involving antisocial behavior is particularly interesting, symbolizing the need for methods such as those illustrated in this article. Patterson’s (1993) work actually investigated growth in a composite variable created within the study. Specifically, the same instrument was administered to each youth, the youth’s parent, and the youth’s teacher at each of four time points (Grades 4, 6, 7, and 8) to get information about potential changes in the youth’s antisocial behavior. The three ratings at each time point were then summed to create an index of antisocial behavior for use in first-order latent growth modeling. Taking such an approach implicitly treats antisocial behavior as an emergent construct rather than a latent construct (Bollen & Lennox, 1991; Cohen, Cohen, Teresi, Marchi, & Velez, 1990; Cole, Maxwell, Arvey, & Salas, 1993), which is questionable given the nature of the antisocial behavior variable system under investigation. In other words, it would seem more theoretically defensible to regard antisocial behavior as a latent construct that manifests itself in the youth’s self-report ratings, as well as in those ratings by the parent and teacher. Treating antisocial behavior as a latent construct would also have the added advantage of avoiding a composite that incorporates the variables’ measurement errors, thereby allowing the theoretically error-free construct to be inferred; this construct could then be incorporated into a latent growth model, utilizing a second-order factor structure to investigate change in a latent construct \(\eta\) across time. Such a modeling of growth at the latent level is denoted in this article as a second-order latent growth model and has been discussed previously by McArdle (1988) as a “curve-of-factors model” as well as by Tisak and Meredith (1990) as a “latent variable longitudinal curve model.”

To elaborate briefly, the general notion of a second-order factor model is one familiar to researchers using oblique confirmatory factor analysis, where the question often arises as to the source of those first-order factors’ obliqueness. Researchers may choose to model first-order factors as dependent on one or more exogenous second-order factors, with the latter thus having only the first-order factors as indicators (i.e., no measured variable indicators). A second-order factor is therefore serving to explain variance in, and covariance among, first-order factors. This idea also exists within a growth modeling context, stemming largely
from work by McArdle (1988; see Duncan & Duncan, 1996, for examples in the context of adolescent substance abuse). McArdle’s factor-of-curves model, for example, uses data from the same individuals over time to model first-order intercept and growth factors for different measured variables as dependent on a second-order intercept factor (affecting only first-order intercept factors) and on a second-order growth factor (affecting only first-order growth factors). In such a model, covariation among latent intercepts is assumed to be explained by the common latent intercept construct, whereas covariation among latent growth rates is assumed to be explained by a common latent growth construct. McArdle also described a curve-of-factors model, which attempts to explain means, variances, and covariances of first-order factors by using exogenous second-order factors representing, for example, latent intercept and latent growth rates. The purpose of this article is to provide a detailed illustration of this latter underutilized longitudinal method of analysis. Its technical basis is as follows.

For change being assessed across $t$ time points, let $\eta_j$ be a latent construct indicated at time $j$ by $k$ measured variables $Y_{ij}$ ($i = 1, \ldots, k$). That is, $y = \tau + \Lambda \eta + \varepsilon$, where the vector $y$ contains $t$ sets of values across time for $k$ $Y$ variables, $\tau$ is a vector of variable intercepts, $\Lambda$ is a matrix of loadings relating each $\eta_j$ construct to its measured variable indicators, $\eta$ is a vector of the $\eta_j$ constructs, and $\varepsilon$ is a vector of random normal errors. So far, this is simply a conventional first-order confirmatory factor model with its mean structure modeled simultaneously. As for modeling growth in the $\eta_j$ constructs, it can be described by $\eta = \Gamma \xi + \zeta$, where $\Gamma$ is a matrix of second-order factor loadings reflecting the hypothesized growth pattern underlying the $\eta_j$ constructs, $\xi$ is a vector of exogenous latent factors capturing the facets of growth being modeled (e.g., initial amount and rate of change), and $\zeta$ is a vector of random normal disturbances in the first-order $\eta_j$ constructs. Similar to the first-order growth model presented earlier, a growth hypothesis could posit $\xi = [\alpha \beta]'$, where $\alpha$ is an intercept factor representing the true initial amount of $\eta$ and $\beta$ is a slope factor representing the true rate of change in $\eta$ over time. Assuming (for simplicity) that indicators of the $\eta$ construct are measured at four equal-interval time points, a test for linear growth in the construct could be conducted by fitting second-order factor loadings $\Gamma$ equivalent to those in the first-order loading matrix $\Lambda$ shown previously. Readers interested in an exposition of moments for the latent constructs in $\eta$, as well as for the measured variables in $y$, may consult Appendix A.

EXAMPLES

Although, as mentioned, Patterson’s (1993) work involving antisocial behavior symbolizes the need for a second-order growth model, sufficient information was not presented in that study to conduct a latent variable longitudinal curve analysis.
Thus, to illustrate second-order latent growth models, cases were randomly drawn for 791 girls and 764 boys from the National Education Longitudinal Survey of 1988 (NELS: 88) data set, sponsored by the National Center for Educational Statistics, U.S. Department of Education (see Ingels et al., 1994). The cases drawn had no missing data for the variables of interest, which were responses to items relating to self-concept. Specifically, at 8th, 10th, and 12th grades, these students were asked to respond to the following statements: “On the whole I feel good about myself”; “I feel I am a person of worth”; and “On the whole I am satisfied with myself.” (These items are BYS44A, BYS44D, and BYS44H from the base year [eighth grade], F1S62A, F1S62D, and F1S62H from the first follow-up [10th grade], and F2S66A, F2S66D, and F2S66H from the second follow-up [12th grade]. The numbers associated with these items are used as variable labels in Figures 1 and 2.) Students’ ratings to these stems were made on a 4-point scale ranging from 1 (strongly agree) to 4 (strongly disagree); for this example, however, all responses were reflected so that higher numbers indicated greater agreement and hence stronger self-concept. Responses to these three items, summarized in Table 1 for girls and boys separately, may serve as indicators of a latent self-concept construct, the linear growth in which may be modeled across the three equal-interval time points.

Covariance and Mean Structures

The second-order latent growth model depicted in Figure 1 was first fitted just to the female students’ data using maximum likelihood estimation in EQS 5.7 (Bentler, 1998); relevant unstandardized results are shown in the figure, and EQS code appears in Appendix B. (Information regarding a model for male students is mentioned later.)

Given that we wish to estimate the means for the intercept construct $\alpha$ and the growth construct $\beta$, a mean structure must be estimated simultaneously along with the covariance structure. As seen in Figure 1, this is represented symbolically by regressing the second-order factors $\alpha$ and $\beta$ on the unit vector 1. This also leaves second-order construct variance information to reside completely within the second-order disturbance variances. Regressing the first-order self-concept factors on 1, thereby yielding latent intercepts, is not necessary because the expected value for $\eta$ is $E[\eta] = \Gamma E[\xi]$; that is, amounts of latent self-concept are theoretically reproducible from the initial amount $\alpha$ and rate of change $\beta$ alone without additional intercept terms. However, regression of observed variables on the unit vector is necessary to estimate the vector of intercept terms $\tau$ because the expected value for $y$ depends on $\tau$: $E[y] = \Lambda E[\eta] + \tau$. One exception is as follows: The unit vector 1 was not regressed on the variables used as the scale indicators for the first-order factors. With loadings fixed to 1, the structural equations for these indicator variables alone effectively constitute a first-order growth model; as such no intercepts are necessary.
Before discussing the results, several additional points about this model are worth noting. First, growth was modeled across 8th, 10th, and 12th grades assuming a linear form; for this reason the paths from the $\beta$ (slope) factor were 0, 1, and 2, respectively. This linear form may or may not be a realistic representation of growth for self-concept; it may actually be of a different functional nature (e.g., quadratic or logarithmic). Nonetheless, given that growth processes are generally measured over a fairly restricted portion of the span of development, even complex growth patterns are often well approximated by simpler models such as this.

\[
\Psi = \begin{bmatrix}
.254^* \\
0 \\
0 \\
\end{bmatrix}, \quad \Psi_{\alpha\beta} = \Phi = \begin{bmatrix}
\zeta_\alpha & \zeta_\beta \\
.256^* & .048 \\
.1301^* & .075 \\
\end{bmatrix}^* \quad p < .05
\]

**FIGURE 1** Covariance and mean structure for linear growth in latent self-concept for 791 female students. For all pairs of parameter estimates, the female students’ value is presented first. Also, underlined numbers were fixed to the values displayed.
### TABLE 1
Descriptive Information for 791 Girls (Below Diagonal) and 764 Boys (Above Diagonal)

<table>
<thead>
<tr>
<th></th>
<th>8th Grade</th>
<th></th>
<th>10th Grade</th>
<th></th>
<th>12th Grade</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>44A</td>
<td>44D</td>
<td>44H</td>
<td>62A</td>
<td>62D</td>
<td>62H</td>
<td>66A</td>
<td>66D</td>
<td>66H</td>
<td>Y</td>
</tr>
<tr>
<td>44A</td>
<td>.360</td>
<td>.114</td>
<td>.098</td>
<td>.034</td>
<td>.009</td>
<td>–.010</td>
<td>.014</td>
<td>3.38</td>
<td>1.10</td>
<td>.91</td>
<td>.93</td>
</tr>
<tr>
<td>44D</td>
<td>.460</td>
<td>.069</td>
<td>.109</td>
<td>.040</td>
<td>.077</td>
<td>.068</td>
<td>.088</td>
<td>3.28</td>
<td>2.12</td>
<td>.93</td>
<td>.93</td>
</tr>
<tr>
<td>44H</td>
<td>.523</td>
<td>.075</td>
<td>.049</td>
<td>.049</td>
<td>.047</td>
<td>.048</td>
<td>.078</td>
<td>3.26</td>
<td>1.08</td>
<td>.91</td>
<td>.93</td>
</tr>
<tr>
<td>62D</td>
<td>.147</td>
<td>.152</td>
<td>.831</td>
<td>.93</td>
<td>.193</td>
<td>.213</td>
<td>.198</td>
<td>3.08</td>
<td>1.36</td>
<td>.91</td>
<td>.93</td>
</tr>
<tr>
<td>62H</td>
<td>.134</td>
<td>.163</td>
<td>.119</td>
<td>.137</td>
<td>.193</td>
<td>.206</td>
<td>.203</td>
<td>2.90</td>
<td>1.43</td>
<td>.91</td>
<td>.93</td>
</tr>
<tr>
<td>66A</td>
<td>.095</td>
<td>.140</td>
<td>.123</td>
<td>.137</td>
<td>.925</td>
<td>.938</td>
<td>.938</td>
<td>2.72</td>
<td>2.12</td>
<td>.91</td>
<td>.93</td>
</tr>
<tr>
<td>66D</td>
<td>.105</td>
<td>.141</td>
<td>.131</td>
<td>.131</td>
<td>.866</td>
<td>.936</td>
<td>.936</td>
<td>2.62</td>
<td>2.15</td>
<td>.91</td>
<td>.93</td>
</tr>
<tr>
<td>66H</td>
<td>.076</td>
<td>.138</td>
<td>.125</td>
<td>.141</td>
<td>.888</td>
<td>.839</td>
<td>.839</td>
<td>2.56</td>
<td>2.10</td>
<td>.91</td>
<td>.93</td>
</tr>
<tr>
<td>( \bar{Y} )</td>
<td>3.10</td>
<td>3.00</td>
<td>2.96</td>
<td>2.81</td>
<td>2.88</td>
<td>2.91</td>
<td>2.80</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>.93</td>
<td>1.16</td>
<td>1.33</td>
<td>1.40</td>
<td>1.60</td>
<td>1.67</td>
<td>1.65</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note.** Responses to all items were reflected so that larger numbers indicated greater agreement (and hence stronger self-concept).
(Willett & Sayer, 1994). Second, the same variable was chosen as the scale indicator for each first-order factor, whereas corresponding variables whose loadings were free had those loadings constrained to be equal; this ensured a comparable definition of the self-concept construct over time (referred to as “stationarity” by Tisak and Meredith, 1990). Third, the errors in corresponding variables were allowed to covary across time as discussed, for example, by Loehlin (1998); these covariances were omitted from Figure 1 for purposes of clarity. Fourth, as is commonly done, the $\alpha$ (intercept) and $\beta$ (slope) factors were allowed to covary; this accommodates the realistic possibility that students’ self-concept development over time is related to where they start at the initial time point (in this case, eighth grade). However, because both factors are modeled as dependent upon the unit vector 1, this desired covariance is carried by a covariance between the factors’ disturbances $\zeta_\alpha$ and $\zeta_\beta$. Fifth, intercepts for corresponding variables at different time points were constrained to be equal, reflecting the fact that change over time in a given variable should start at the same initial point. Thus, from the nine measured variable means, only four parameters are estimated uniquely (two variable intercepts and two factor means), making the means portion of this model overidentified. Finally, considering the covariance structure model alone, the 45 pieces of information (i.e., observed variable variances and covariances) are used to estimate 26 parameters: 2 second-order disturbance variances, 1 second-order disturbance covariance, 3 first-order disturbance variances, 2 unique loadings, and 18 error variances and covariances. The covariance structure is thus overidentified with $19 \text{ df}$; however, the second-order portion of this model is just-identified, estimating six parameters (2 second-order disturbance variances, 1 second-order disturbance covariance, and 3 first-order disturbance variances) from the variance–covariance information provided by the 3 first-order factors.

This model, which has $24 \text{ df}$ overall (5 from the mean structure and 19 from the covariance structure), fit the female students’ data extremely well: $\chi^2 = 35.374, p = .063$; comparative fit index (CFI) = .998, standardized root mean squared residual (RMSR) = .006, and root mean squared error of approximation (RMSEA) = .025 (90% confidence interval [CI]—.000, .041). The parameters of greatest interest, those relating to the latent growth structure, appear in Figure 1. Note that in the $\Psi_{\alpha\beta}$ matrix there is a statistically significant ($p < .05$) amount of variability in the intercept factor (captured in its disturbance), indicating that the female students do not start with the same latent level of self-concept (i.e., $\alpha$). Linear growth in latent self-concept from that initial amount (i.e., $\beta$), however, tends to be somewhat comparable across girls as indicated by a variance in the slope factor (captured in its disturbance) that was not statistically significantly different from 0. Given this similarity in growth rate, it follows logically that female students’ degree of latent growth would be unrelated to their initial level of self-concept as indicated by a nonsignificant interfactor (technically, interdisturbance) covariance.
As for parameters associated with the means portion of the model, the variables’ intercepts are shown but are not terribly interesting; the paths of key interest are those from the unit vector 1 to $\alpha$ and $\beta$, representing estimates of these constructs’ means. The initial amount of self-concept is estimated as 3.102; as expected, this is effectively equivalent to the sample mean for the eighth-grade self-concept scale indicator 44A shown in Appendix B. More interesting, however, is the estimated growth rate in latent self-concept per 2-year interval. The numerical value of $-0.122$, which is tied to the units of the self-concept scale indicators, is perhaps less important than the fact that it is statistically significantly ($p < .05$) less than 0. This indicates that the acceptably fitting model of linearity is in fact linearly decreasing, thereby representing an average tendency for female students’ self-concept to decline over the time period in question. This is merely an average, though, and not necessarily indicative of the trend for all girls. An estimate of the standard deviation of slopes may be computed as the square root of the slope factor (disturbance) variance, $(0.075)^{1/2} = 0.274$; the fact that this is larger than the mean of the $\beta$ slope factor itself may imply that rates of change for some girls are positive whereas for others they are negative.

Finally, before proceeding to the next example, it should be noted that the previous model was also fitted to the data for 764 boys similarly drawn from the NELS:88 database. Without detailing all parameters, acceptable fit was also achieved: $\chi^2 = 69.265, p < .001$; CFI = .992, standardized RMSR = .007, and RMSEA = .050 (90% CI—.036, .063). One may wonder why these data were not simply combined with those of the 791 girls and run as a single model on the combined sample of 1,555 students. The answer is that mean gender differences on the measured variables may artificially inflate or deflate observed covariances, thereby leaving the model to be a poor representation of variable relations for either gender group even if the relations within each gender group had been similar. Further, even if gender had been controlled (either through partialling a gender dummy variable from the data or incorporating that variable as a predictor in the model), the possibility still exists that fundamentally different relations among variables and constructs exist. In the next section, a multigroup strategy illustrates the simultaneous assessment and comparison of models for boys and girls, including both covariance and mean structures.

**Multisample Second-Order Latent Growth Model**

As suggested by, for example, Tisak and Meredith (1990), models for multiple samples may be investigated simultaneously. This facilitates the examination of model invariance, including a comparison of key construct parameters for the second-order growth constructs. Given that this model fits both female and male students’ data satisfactorily when examined separately, a multisample model was im-
posed on the data with cross-group loading and intercept constraints for measured variables. Specifically, in addition to the first measured variable serving as the scale indicator for all first-order factors in both groups (an implicit constraint), all loadings for the second variable were constrained to be equal on all factors and constrained to be equal across groups; a similar set of constraints was imposed on the loadings of the third measured indicator. Further, in addition to the scale indicator variable for each factor having a zero intercept in both groups (an implicit constraint), all intercepts for the second variable were constrained to be equal on all factors and constrained to be equal across groups; a similar set of constraints was imposed on the intercepts of the third measured indicator. Also noteworthy is the fact that, unlike most multisample means models, this analysis does not require one group to serve as a reference group with its construct means set to zero; rather, as both means models are independently overidentified, both construct means in both groups could be freely estimated. This multisample covariance and mean structure model, which has 52 $df$ overall, fit quite well: $\chi^2 = 124.123, p < .001$; CFI = .993, standardized RMSR = .011, and RMSEA = .042 (90% CI—.033, .052). (Note that the RMSEA and its confidence interval reflect the multisample model correction suggested by Steiger [1998].) EQS code for this example appears in Appendix B.

The portion of interest in the multisample model, that relating the second-order intercept and growth factors to the first-order self-concept constructs, is presented in Figure 2.

Note that the unconstrained, unstandardized parameter estimates appear for both the male and female students (with the female student estimate being the first presented in each pair). Five parameters of primary interest are the 2 second-order factor means, 1 second-order factor covariance, and to a lesser extent the 2 second-order factor variances. To assess the statistical significance of the difference between each pair of parameter estimates, five additional models were run in which one and only one pair of key parameters was constrained to be equal across groups in addition to preexisting loading and intercept constraints for measured variables. The difference between each model’s $\chi^2$ and that of the base multisample model with no second-order parameter constraints provided a 1-$df$ test of equality for each parameter pair. Results of such difference tests indicated that, of the five key parameter pairs, only the two pairs of construct means contained statistically significant differences at the .05 level. Specifically, male students’ second-order intercept factor mean of 3.373 was statistically significantly higher than the female students’ value of 3.127 ($p < .001$), indicating that boys tend to start with a higher latent self-concept in eighth grade. Also, male students’ second-order growth factor mean of −.310 was statistically significantly lower (i.e., even more negative) than the female students’ value of −.122 ($p < .001$), indicating that the average decline in latent self-concept over this time period for male students is sharper than that for the female students.
Although statistical significance has been established for the difference between male and female students’ latent intercept and latent growth construct means, the magnitude of the difference from a practical perspective is not immediately apparent in the reported results. Fortunately, one may apply the principles of standardized effect size estimates (Cohen, 1988) to latent mean differences, as has been done in other applied literature (e.g., Aiken, Stein, & Bentler, 1994; Dukes, Ullman, & Stein, 1995). Specifically, for the two-group case a latent standardized effect size estimate \( \Delta \) may be derived as

\[
\Delta = \frac{\hat{\kappa}_i - \hat{\kappa}_j}{\sqrt{\hat{\psi}_\xi}},
\]

where \( \hat{\kappa}_i \) is the \( i \)th group’s construct mean and \( \hat{\psi}_\xi \) is the pooled average of the two unconstrained construct (disturbance) variances using sample sizes as weights (or an equality constrained construct disturbance variance). The pooled variances for the latent intercept and growth constructs are

\[
\text{pooled variance for intercept} = \frac{791(.257) + 764(.079)}{1555} = .170
\]

\[
\text{pooled variance for growth} = \frac{791(.076) + 764(.236)}{1555} = .155
\]

Using the pooled values, latent standardized effect size estimates for the latent inter-

---

**FIGURE 2** Second-order covariance and mean structures for 791 female students and 764 male students. Errors in corresponding variables were allowed to covary across time; these are omitted from the figure only for purposes of clarity. In addition, underlined numbers were fixed to the values displayed.
cept and growth constructs are $\hat{\Delta}_\alpha = .597$ and $\hat{\Delta}_\beta = .478$, respectively. Applying Cohen’s (1988) guidelines (which, admittedly, were offered for measured variable rather than latent variable differences), both latent construct differences would constitute roughly medium effect sizes. That is, on average male students start about half a standard deviation higher in latent self-concept in eighth grade, and on average decline in latent self-concept at a rate roughly half a standard deviation more swiftly.

**COMMENTS AND CONCLUSIONS**

The methods described in this article are not without their assumptions. In addition to common structural equation modeling assumptions, it is also assumed that growth can be described by a function (e.g., linear), that this functional form holds for all members of the population, and that variability in individual coefficients associated with this function is distributed normally. Robustness of growth models to violations of these assumptions is not yet well addressed within the structural equation modeling literature.

In these examples a linear functional form was chosen and fit quite well; still, this does not necessitate that latent change is linear. Growth could actually proceed along a different function, such as quadratic or logarithmic, and loadings may generally be chosen to reflect such other developmental trajectories. Nonetheless, as mentioned previously, given that growth processes have been assessed in these examples over a fairly restricted portion of the span of development, a more complex growth pattern may still be well approximated by this simple model (Willett & Sayer, 1994). With only three time points, these examples were limited to investigating only one growth component by the implicit constraints of model identification (unless further constraints on error variances, such as fixing them to zero to emulate a repeated measure analysis of variance design, is desirable). That is, a quadratic growth factor, for example, could not be incorporated into $\xi$ in addition to $\alpha$ and $\beta$ without further constraints to the model. However, if data had also been gathered 2 years after 12th grade, the limits of model identification would have permitted a second-order latent growth model with $\xi = [\alpha \beta \chi]'$, where $\chi$ is, say, a quadratic growth factor. This model could be tested by imposing, for example, the following second-order loadings:

$$
\Gamma = \begin{bmatrix}
1 & 1 & 1 & 1 \\
0 & 1 & 2 & 3 \\
0 & 1 & 4 & 9 \\
\end{bmatrix}
$$

Such extensions are discussed by Stoolmiller (1995), and an excellent example of nonlinear growth is illustrated by Stoolmiller et al. (1993).
Also noteworthy is the choice of $\alpha$ as representing the initial amount of the latent construct. It could, in fact, be used to represent the amount of the construct at any time point (even one for which data were not gathered). For example, to reflect the amount of latent self-concept in Grade 10 (if that were some theoretically interesting developmental juncture) the paths from $\beta$ to the first-order constructs could simply be changed to $-1, 0,$ and $1$, respectively. Such a change would also necessarily alter the amount of covariation between this new $\alpha$ and $\beta$, so care should be used in interpretation of constructs and their associated parameters.

Further, as with first-order growth models, external predictors of intercept and slope may be incorporated. These predictors may be latent constructs with their own measured indicators, or measured variables (e.g., an index of socioeconomic status). One particular type of measured external predictor includes dummy variables indicating group membership, such as gender, or ethnic group, or even treatment group. These predictors may be used to infer mean differences in intercept and growth constructs without necessarily modeling the mean structure and has been suggested as an alternative to multisample mean structure models (Muthén, 1989); see work by Duncan and Duncan (1995) for an example. As mentioned previously, however, conducting multisample modeling does allow for the elucidation of potentially different relations among variables and constructs. Whereas incorporating group membership variables leads to a single measurement and structural model for all groups, multisample models facilitate invariance investigations. With specific regard to second-order growth models as presented in this article, invariance across groups may be of interest not just at a structural level, but also with respect to the first-order construct’s measurement within and between groups.

In sum, LCA methods in general comprise a versatile tool in the researcher’s longitudinal analysis tool belt. They facilitate the testing of linear, nonlinear, or a combination of growth trajectories over time, as well as allow the researcher to investigate the effects of external variables on the growth system of interest. More particularly, this article illustrates an extension of common LCA to model growth in latent constructs that are indicated by multiple measured variables, both for single sample and multisample scenarios. Such methods offer the advantage of creating a theoretically error-free construct for growth modeling rather than using error-laden variables or their composites. It is our hope that these illustrations will help bring these methods to applied researchers working toward a better understanding of longitudinal development in their respective fields.

REFERENCES


APPENDIX A
FIRST AND SECOND MOMENTS WITHIN FIRST-ORDER AND SECOND-ORDER LATENT GROWTH MODELS

First-Order Growth Model

Given the expectation $E[\delta] = \mathbf{0}$, the model-implied first moment is $\mu = E[x] = \Lambda E[\xi] = \Lambda \kappa$, where $\kappa$ is a vector of factor means. As for the model-implied second moment $\Sigma$, assuming that $E[(\xi - \kappa) \delta'] = \mathbf{0}$ leads to $\Sigma = E[(x - \Lambda \kappa)(x - \Lambda \kappa)'] = \Lambda \Phi \Lambda' + \Theta$, where $\Phi$ is the variance–covariance matrix of the factors in $\xi$ and $\Theta$ is the variance–covariance matrix of the errors in $\delta$.

Second-Order Growth Model

In the second-order portion of the model, the expectation $E[\zeta] = \mathbf{0}$ yields the model-implied first moment $\nu = E[\eta] = \Gamma E[\xi] = \Gamma \kappa$, where $\kappa$ is a vector of means for the factors in $\xi$ facilitating the assessment of growth. As for the model-implied second moment $\Omega$, assuming $E[(\xi - \kappa) \zeta'] = \mathbf{0}$ leads to $\Omega = E[(\eta - \nu)(\eta - \nu)'] = \Gamma \Phi \Gamma' + \Psi$, where $\Phi$ is the variance–covariance matrix of the factors in $\xi$ and $\Psi$ is the variance–covariance matrix of the disturbances in $\zeta$. In the first-order portion of the
model, the expectation \( E[\varepsilon] = 0 \) yields the model-implied first moment \( \mu = E[y] = \Lambda E[\eta] + \tau = \Lambda \nu + \tau = \Lambda \Gamma \kappa + \tau \), where \( \Lambda \) represents the matrix of first-order factor loadings and \( \tau \) is a vector of variable intercepts. Similarly, assuming \( E[(\eta - \nu)\varepsilon'] = 0 \), the model-implied second moment is \( \Sigma = E[(y - E(y))(y - E(y))'] = \Lambda \Omega \Lambda' + \theta \) where \( \theta \) is the variance–covariance matrix of the errors in \( \varepsilon \). If one additionally assumes \( E[(\xi - \kappa)\varepsilon'] = 0 \) and \( E[\zeta \varepsilon] = 0 \), this second moment becomes \( \Sigma = \Lambda (\Gamma \Phi \Gamma' + \Psi) \Lambda' + \theta = \Lambda (\Gamma \Phi \Gamma') \Lambda' + \Lambda \Psi \Lambda' + \theta \).

APENDIX B

EQS Syntax for Single Sample and Multisample Models

<table>
<thead>
<tr>
<th>EQS Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>V measured variable</td>
</tr>
<tr>
<td>F factor (latent variable)</td>
</tr>
<tr>
<td>V999 unit constant (1) (not a variable)</td>
</tr>
</tbody>
</table>

EQS syntax for modeling female students’ covariance and mean structure for linear growth in latent self-concept (see Figure 1):

/TITLE
Covariance and means model for females’ self-concept
/SPECIFICATIONS
  cases=791; variables=9; matrix=cor; method=ml; analysis=moment;
/LABELS
  V1 = 44A; V2 = 44D; V3 = 44H; V4 = 62A; V5 = 62D; V6 = 62H;
  V7 = 66A; V8 = 66D; V9 = 66H; F1 = SC8; F2 = SC10; F3 = SC12;
  F4 = ALPHA; F5 = BETA;
/EQUATIONS
  V1 = 1F1 + 0V999 + E1;
  V2 = *F1 + *V999 + E2;
  V3 = *F1 + *V999 + E3;
  V4 = 1F2 + 0V999 + E4;
  V5 = *F2 + *V999 + E5;
  V6 = *F2 + *V999 + E6;
  V7 = 1F3 + 0V999 + E7;
  V8 = *F3 + *V999 + E8;
  V9 = *F3 + *V999 + E9;
  F1 = 1F4 + 0F5 + D1;
  F2 = 1F4 + 1F5 + D2;
F3 = 1F4 + 2F5 + D3;
F4 = *V999 + D4;
F5 = *V999 + D5;

/VARIANCES
E1 to E9 =*;
D1 to D5 =*;

/COVARIANCES
D4,D5 =*;
E1,E4 =*; E1,E7 =*; E4,E7 =*;
E2,E5 =*; E2,E8 =*; E5,E8 =*;
E3,E6 =*; E3,E9 =*; E6,E9 =*;

/CONSTRAINTS
(V2,F1) = (V5,F2) = (V8,F3);
(V3,F1) = (V6,F2) = (V9,F3);
(V2,V999) = (V5,V999) = (V8,V999);
(V3,V999) = (V6,V999) = (V9,V999);

/MATRIX
1.000
.460 1.000
.523 .458 1.000
.201 .147 .173 1.000
.147 .143 .152 .818 1.000
.134 .132 .163 .835 .831 1.000
.095 .055 .140 .119 .123 .137 1.000
.105 .091 .141 .102 .131 .131 .866 1.000
.076 .051 .138 .098 .125 .141 .888 .839 1.000

/STANDARD DEVIATIONS
0.93 1.10 1.16 1.33 1.42 1.40 1.60 1.67 1.65

/MEANS
3.10 3.17 3.00 2.96 2.97 2.81 2.88 2.91 2.80

/PRINT
fit = all;

/EQS syntax for modeling both sexes’ latent self-concept growth (see Figure 2):

/TITLE
Multisample model for males’ and females’ self-concept (group 1 = females)
/SPECIFICATIONS
cases = 791; variables = 9; matrix = cor; method = ml; analysis = moment; groups = 2;
/**LABELS
V1 = 44A; V2 = 44D; V3 = 44H; V4 = 62A; V5 = 62D; V6 = 62H;
V7 = 66A; V8 =66D; V9 = 66H; F1 = SC8; F2 = SC10; F3 = SC12;
F4 = ALPHA; F5 = BETA;
*/

/**EQUATIONS
V1 = 1F1 + 0V999 + E1;
V2 = *F1 + *V999 + E2;
V3 = *F1 + *V999 + E3;
V4 = 1F2 + 0V999 + E4;
V5 = *F2 + *V999 + E5;
V6 = *F2 + *V999 + E6;
V7 = 1F3 + 0V999 + E7;
V8 = *F3 + *V999 + E8;
V9 = *F3 + *V999 + E9;
F1 = 1F4 + 0F5 + D1;
F2 = 1F4 + 1F5 + D2;
F3 = 1F4 + 2F5 + D3;
F4 = *V999 + D4;
F5 = *V999 + D5;
*/

/**VARIANCES
E1 to E9 =*;
D1 to D5 =*;
*/

/**COVARIANCES
D4,D5 =*;
E1,E4 =*; E1,E7 =*; E4,E7 =*;
E2,E5 =*; E2,E8 =*; E5,E8 =*;
E3,E6 =*; E3,E9 =*; E6,E9 =*;
*/

/**CONSTRAINTS
(1,V2,F1) = (1,V5,F2) = (1,V8,F3);
(1,V3,F1) = (1,V6,F2) = (1,V9,F3);
(1,V2,V999) = (1,V5,V999) = (1,V8,V999);
(1,V3,V999) = (1,V6,V999) = (1,V9,V999);
*/

/**MATRIX
1.000
.460 1.000
.523 .458 1.000
.201 .147 .173 1.000
.147 .143 .152 .818 1.000
.134 .132 .163 .835 .831 1.000
.095 .055 .140 .119 .123 .137 1.000
.105 .091 .141 .102 .131 .131 .866 1.000
.076 .051 .138 .098 .125 .141 .888 .839 1.000
*/
Multisample model for male and female students’ self-concept (group 2 = males)

EQUATIONS
V1 = 1F1 + 0V999 + E1;
V2 = *F1 + *V999 + E2;
V3 = *F1 + *V999 + E3;
V4 = 1F2 + 0V999 + E4;
V5 = *F2 + *V999 + E5;
V6 = *F2 + *V999 + E6;
V7 = 1F3 + 0V999 + E7;
V8 = *F3 + *V999 + E8;
V9 = *F3 + *V999 + E9;
F1 = 1F4 + 0F5 + D1;
F2 = 1F4 + 1F5 + D2;
F3 = 1F4 + 2F5 + D3;
F4 = *V999 + D4;
F5 = *V999 + D5;

VARIANCES
E1 to E9 = *;
D1 to D5 = *;

COVARIANCES
D4,D5 = *;
E1,E4 = *; E1,E7 = *; E4,E7 = *;
E2,E5 = *; E2,E8 = *; E5,E8 = *;
E3,E6 = *; E3,E9 = *; E6,E9 = *;

CONSTRAINTS
(2,V2,F1) = (2,V5,F2) = (2,V8,F3) = (1,V2,F1) = (1,V5,F2) = (1,V8,F3);
(2,V3,F1) = (2,V6,F2) = (2,V9,F3) = (1,V3,F1) = (1,V6,F2) = (1,V9,F3);
(2,V2,V999) = (2,V5,V999) = (2,V8,V999) = (1,V2,V999) = (1,V5,V999) = (1,V8,V999);
(2,V3,V999)=(2,V6,V999)=(2,V9,V999)=(1,V3,V999)=(1,V6,V999)=(1,V9,V999);

/MATRIX
1.000
.360 1.000
.514 .505 1.000
.114 .089 .034 1.000
.069 .109 .040 .810 1.000
.075 .102 .049 .775 .745 1.000
.009 .077 .047 .210 .193 .193 1.000
-.010 .068 .048 .197 .213 .206 .925 1.000
.014 .088 .078 .200 .198 .203 .938 .936 1.000

/STANDARD DEVIATIONS
0.76 1.10 0.91 1.26 1.36 1.43 2.12 2.15 2.10

/MEANS
3.38 3.28 3.26 3.14 3.08 2.90 2.72 2.62 2.56

/PRINT
fit = all;

/END