The (Sometimes Harsh) Reality of Longitudinal Student Achievement Modeling

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Outline

- Background for case study
- Challenges:
  - Variance partitioning
  - Missing data
  - Structuring hypothesis about cumulative effects
  - Model estimation with nuisance correlations
- Closing thoughts
Case Study: The RAND Mosaic II Project

- Study goal: Examine relationships between “reform-oriented teaching practices” and student mathematics and science achievement
  - Hands-on and investigative activities
  - Exploration of students’ thinking

- Mosaic I: Found small positive effects of one year of exposure to reform teaching

- Mosaic II: Improved methods for assessing reform teaching, additional consideration of open-ended assessments, followed students through three years of exposure
## Data Structure

Basic design replicated in 5 cohorts

**MC** = Multiple Choice \quad **OE** = Open-Ended

<table>
<thead>
<tr>
<th>Cohort 1</th>
<th>Mathematics</th>
<th>Grades 3-5</th>
<th>SAT9 MC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohort 2</td>
<td>Mathematics</td>
<td>Grades 7-9</td>
<td>SAT9 MC</td>
</tr>
<tr>
<td>Cohort 3</td>
<td>Mathematics</td>
<td>Grades 6-8</td>
<td>SAT9 MC (PR,PS) + OE</td>
</tr>
<tr>
<td>Cohort 4</td>
<td>Science</td>
<td>Grades 3-5</td>
<td>SAT9 MC + OE</td>
</tr>
<tr>
<td>Cohort 5</td>
<td>Science</td>
<td>Grades 6-8</td>
<td>SAT9 MC + OE</td>
</tr>
</tbody>
</table>

- MC administered in Years 1, 2 and 3 in all cohorts
- OE administered in some years for some cohorts
Specific Data Elements

- **Student Level:**
  - Assessment scores (usually scaled) from Years 1-3
  - Assessment scores from districts and/or state tests from “Year 0”, the year prior to the study
  - Background variables generally including: race, FRL, LEP, special education, gifted, and age (used to proxy for “behind cohort”)
  - Links to teachers in Years 1-3

- **Teacher Level:**
  - Measures of teaching practices and background characteristics obtained from multiple methods (surveys, lesson logs, instructional vignettes)
Challenges:

- Variance partitioning
- Missing data
- Structuring hypothesis about cumulative effects
- Model estimation with nuisance correlations
Exploratory Analyses on Scores Suggests Small Explainable Variance

- For each cohort, year, and outcome, decompose variance of level scores into four sources:
  1. Background variables and Year 0 scores *within teachers* (“X within”)
  2. Unexplained variance *within teachers*
  3. Aggregate background variables and year 0 scores *between teachers* (“X between”)
  4. Unexplained variance *between teachers*

- $(3 + 4)$ bounds $R^2$ of main effect of a teacher-level predictor with respect to achievement levels

- Ideally: $(3 + 4)$ is big and $4 \gg 3$

- Empirically: $(3 + 4) \ll (1 + 2)$ and 4 is generally tiny
Variance Decomposition of Level Scores (Example)

Cohort 1

- X within
- Residual within
- X between
- Residual between

% Total Variance

G3  G4  G5

October 31, 2005-8
Variance Decomposition of Level Scores: Mathematics

Cohort 1

Cohort 2

Cohort 3 (MC TO)

Cohort 3 (MC PR)

Cohort 3 (MC PS)

Cohort 3 (OE)

October 31, 2005- 9
Variance Decomposition of Level Scores: Science

Cohort 4 (MC)

Cohort 4 (OE)

Cohort 5 (MC)

Cohort 5 (OE)
Can Strong Relationships Between Aggregate X and Teacher Predictors Leave Hope of Big Effects?

Not Really

![Graph showing the relationship between adjusted R-squared and index for a teacher-level variable. The graph indicates a trend but no clear hope of big effects.]
Challenges:

- Variance partitioning
- Missing data
- Structuring hypothesis about cumulative effects
- Model estimation with nuisance correlations
Missing Data Compounds Quickly in Longitudinal Studies

- Many levels of missing data:
  - Student test scores (mobility, absenteeism)
  - Student covariates
  - Student-teacher links
  - Unit-level teacher non-response
  - Item-level teacher non-response

- Compounds roughly geometrically across years

- Particular challenge with longitudinal models and cumulative effects is the need for full information on *exposure history*
Relatively Small Fraction of Students Have Complete Information

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total # Students</td>
<td>2415</td>
<td>5173</td>
<td>3460</td>
<td>1864</td>
<td>7827</td>
</tr>
<tr>
<td>Y1 Scores (%)</td>
<td>71</td>
<td>84</td>
<td>59</td>
<td>63</td>
<td>76</td>
</tr>
<tr>
<td>( \cap ) Y2,Y3 Scores (%)</td>
<td>52</td>
<td>63</td>
<td>24</td>
<td>32</td>
<td>54</td>
</tr>
<tr>
<td>( \cap ) Demographics (%)</td>
<td>52</td>
<td>63</td>
<td>24</td>
<td>32</td>
<td>54</td>
</tr>
<tr>
<td>( \cap ) Y0 Scores (%)</td>
<td>45</td>
<td>54</td>
<td>22</td>
<td>NA</td>
<td>48</td>
</tr>
<tr>
<td>( \cap ) Links to Responders (%)</td>
<td>25</td>
<td>23</td>
<td>15</td>
<td>16</td>
<td>35</td>
</tr>
<tr>
<td>( \cap ) Links to Complete Resp. (%)</td>
<td>19</td>
<td>12</td>
<td>10</td>
<td>11</td>
<td>18</td>
</tr>
</tbody>
</table>
Complete Case Analysis?

- Loss of power
- Lack of generalizability: complete cases are truly a selected sample
Average Scores Conditional on Number of Observed Scores

Cohort 2

Standardized Score vs. # Observed Scores

-1.0  -0.8  -0.6  -0.4  -0.2  0.0  0.2

# Observed Scores

October 31, 2005-16
Average Scores Conditional on Number of Observed Scores

Cohort 2

# Observed Scores

Standardized Score

Year 1
Year 2
Year 3
Calibrating the Information Contained in Missing Scores

- Variance decomposition implies the maximal $R^2$ of a teacher-level predictor is on the order 10% or less.

- For a continuous predictor, corresponds to a Cohen-type effect size of about 0.33.

- In these data, approximately equal to the predictive value of knowing the student had one unobserved score sometime over 4 years.
Moving Forward with Missing Data: Multiple Imputation

Logic of multiple imputation

- Obtain $K$ realizations of missing data $D_{mis}$ by sampling from $p(D_{mis}|D_{obs})$ to create $K$ replicates of $D_{full} = (D_{obs}, D_{mis})$
- Fit models to each $D_{full}$ as if data were fully observed
- Pool estimates and standard errors across model fits to obtain global inferences that account for uncertainty due to $D_{mis}$

Conceptually straightforward

Practical challenge is positing $p(D_{mis}|D_{obs})$ in such a way to maintain fidelity to multivariate structure of observed data
Implemented Multi-Stage Multiple Imputation Procedure

Relied heavily on Schafer’s norm package for R Environment

- Student-level demographics and year 0 scores: use approximate joint distribution of demographics, year 0 scores, future scores and future exposures to teacher-level variables

- Item-level missingness of teacher variables: use approximate joint distribution of item responses and classroom aggregates of student scores and variables

- Missing student-teacher links: create “pseudo-teachers” who receive donated covariate vectors from actual teachers, obtained by informed hotdeck

- Missing test scores from Years 1-3: Imputed “on the fly” during model estimation ("data augmentation")
Missing Data Had Surprisingly Little Leverage on Inferences

- Point estimates for key teacher-level predictors were relatively robust to using complete cases versus sequences of nested sets of students with increasingly poorer observed information.
  - E.g. missing no test scores, versus missing at most 1 test score, versus missing at most 2 test scores.

- Standard errors for estimates under multiple imputation were consistently smaller.
  (rough approximation: $\frac{SE_{MI}}{SE_{complete}} \approx \frac{2}{3}$)

- Reasonable imputations + Complex model that implicitly downweights incomplete cases = Robustness.
Challenges:

- Variance partitioning
- Missing data
- Structuring hypothesis about cumulative effects
- Model estimation with nuisance correlations
An Additive Model For Cumulative Exposure

- General notion: Achievement at time \( t \) is (partially) a function of exposure to practices up to and including time \( t \)

\[
\begin{align*}
Y_{i1} &= f_1(stuff, exposure_{-\infty}, \ldots, exposure_1) + \text{error} \\
Y_{i2} &= f_2(stuff, exposure_{-\infty}, \ldots, exposure_1, exposure_2) + \text{error} \\
Y_{i3} &= f_3(stuff, exposure_{-\infty}, \ldots, exposure_1, exposure_2, exposure_3) + \text{error}
\end{align*}
\]

- Here \( f_t \) is arbitrary but we will be somewhat less ambitious

- Let \( P_{j(i)} \) be the measure of a particular teacher characteristic or teaching practice for teacher \( j \)

\[
\begin{align*}
Y_{i1} &= else + \delta_{11}P_{j(i,1)} + \text{error} \\
Y_{i2} &= else + \delta_{21}P_{j(i,1)} + \delta_{22}P_{j(i,2)} + \text{error} \\
Y_{i3} &= else + \delta_{31}P_{j(i,1)} + \delta_{32}P_{j(i,2)} + \delta_{33}P_{j(i,3)} + \text{error}
\end{align*}
\]
Model Subsumes Some Plausible Alternatives

\[
Y_{i1} = \text{else} + \delta_{11}P_{j(i,1)} + \text{error}
\]
\[
Y_{i2} = \text{else} + \delta_{21}P_{j(i,1)} + \delta_{22}P_{j(i,2)} + \text{error}
\]
\[
Y_{i3} = \text{else} + \delta_{31}P_{j(i,1)} + \delta_{32}P_{j(i,2)} + \delta_{33}P_{j(i,3)} + \text{error}
\]

- \((\delta_{21} = \delta_{31} = \delta_{32} = 0)\): No cumulative effects: exposure this year affects *level score* this year
- \((\delta_{11} = \delta_{21} = \delta_{31})\) and \((\delta_{22} = \delta_{32})\): “Complete” cumulative effects: exposure this year affects *gain score* this year
- General structure with all six parameters unknown allows the data to inform the appropriate degree of accumulation
- Key function of interest is \(\delta_{total} = (\delta_{31} + \delta_{32} + \delta_{33})\)
- Also of interest might be \(\delta_{current} = (\delta_{11} + \delta_{22} + \delta_{33})/3\)
Full Fixed Effects Structure

\[ Y_{i1} = \mu_1 + X_i \beta_1 + Z_{i0} \gamma_1 + \delta_{11} P_{j(i,1)} + \text{error}_{i1} \]

\[ Y_{i2} = \mu_2 + X_i \beta_2 + Z_{i0} \gamma_2 + \delta_{21} P_{j(i,1)} + \delta_{22} P_{j(i,2)} + \text{error}_{i2} \]

\[ Y_{i3} = \mu_3 + X_i \beta_3 + Z_{i0} \gamma_3 + \delta_{31} P_{j(i,1)} + \delta_{32} P_{j(i,2)} + \delta_{33} P_{j(i,3)} + \text{error}_{i3} \]

- \( X_i \): student background variables
- \( Z_{i0} \): student achievement on year 0 assessments
Challenges:

- Variance partitioning
- Missing data
- Structuring hypothesis about cumulative effects
- Model estimation with nuisance correlations
Error Terms Have Structure That Must Be Dealt With

- Correlation within students over time (unmeasured student effects)
- Correlation across students sharing a teacher this year (unmeasured teacher/classroom effects)
- Carry-over effects of past shared classrooms
- Addressing these nuisance correlations is necessary to obtain reasonable standard errors for parameters of interest
Parameterizing The Error Terms

\[ \text{error}_{i1} = \theta_{j(i,1)} + \epsilon_{i1} \]
\[ \text{error}_{i2} = \alpha_{21}\theta_{j(i,1)} + \theta_{j(i,2)} + \epsilon_{i2} \]
\[ \text{error}_{i3} = \alpha_{31}\theta_{j(i,1)} + \alpha_{32}\theta_{j(i,2)} + \theta_{j(i,3)} + \epsilon_{i3} \]

- \( \theta_j \): unobserved “teacher effects”, treated as independent normal random effects with year-specific variance components
- \((\alpha_{21}, \alpha_{31}, \alpha_{32})\) “persistence parameters” that moderate the persistence of past unobserved teacher effects (estimated from data)
- \((\epsilon_{i1}, \epsilon_{i2}, \epsilon_{i3}) \sim N(0, \Sigma)\) independently across students
  - Unstructured covariance proxies for omitted student variables
Complete Model Poses Estimation Challenges

- Off-the-shelf mixed-effects models routines generally not equipped to estimate the complex multiple-membership structure of the random effects with unknown persistence of past teacher effects.

- Built specialized software capitalizing on Bayesian methods (Markov Chain Monte Carlo) for model estimation, which scales to very large datasets.

- However, models with the size of datasets used here probably estimable in WinBugs/OpenBugs (free software for fitting Bayesian models) without much difficulty.
# Cumulative Effects: Mathematics

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<thead>
<tr>
<th>Coh1</th>
<th>Coh2</th>
<th>Coh3</th>
<th>Coh3</th>
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<tbody>
<tr>
<td>Gr 3–5</td>
<td>Gr 7–9</td>
<td>Gr 6–8</td>
<td>Gr 6–8</td>
<td>Gr 6–8</td>
<td>Gr 6–8</td>
<td>Gr 6–8</td>
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<tr>
<td>MC</td>
<td>MC</td>
<td>MC TO</td>
<td>MC PR</td>
<td>MC PS</td>
<td>OE</td>
<td></td>
</tr>
</tbody>
</table>

| Reform Practices | • | • | • | • | • | • |
| Reform Full | • | • | • | • | • | * |
| Average Reform Activities | • | • | • | * | • | ** | • |
| Mixed-ability Groupwork | • | • | • | • | • | • | ** |
| Seatwork | • | • | • | • | • | • | ** |

* Sig. at 0.05  ** Sig. at 0.01
## Cumulative Effects: Science

<table>
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<tr>
<th></th>
<th>Coh4</th>
<th>Coh4</th>
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<td>Average Reform</td>
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<td>Activities</td>
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<td>Mixed–ability</td>
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<td>Groupwork</td>
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<td>Seatwork</td>
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</tbody>
</table>

- **Sig. at 0.01**
- * Sig. at 0.05
Conclusions I

- Increased testing, better data systems, and heightened demand for using data to better the education system are providing growing opportunities for longitudinal analyses.

- However …

- (Variance partitioning)
  - Richness not a panacea for “needle in a haystack”
  - Student background remains strong predictor of achievement.

- (Missing data)
  - Fractured records mount quickly in long and wide multilevel data series.
  - Hard to recover power of (hypothetical) full data, but thoughtful multiple imputation can go a long way.
Conclusions II

- (Cumulative effects)
  - Additive effects are simple and interpretable
  - But do we really believe additivity in such a complex system?

- (Nuisance correlations)
  - Accumulation of unmeasured inputs across changing contexts imparts messy structure to residuals
  - Even a first-order approximation can move the analysis into unfriendly computational territory