EDCI 751 Foundations of Mathematics Education I
Theory and Research on Mathematical Thinking and Learning

Fall 2005
A. Graeber
Mondays 4:15-7 PM
Office: Room 2226F
Office Hours Mondays 3:00-4: 00; Wed. 4:00- 5:00 and arranged.

annagrae@umd.edu
405-7060
Class: Room 2212A

Course Description: Study of mathematical thinking by students at various levels of schooling considered from classic and contemporary theories of learning that are particularly relevant to the study of mathematics. Exploration of what it means to understand mathematics.

Course Overview: The goal of the graduate program at UMCP is to prepare reflective practitioners for classrooms of diverse learners, through research-based inquiry. Doctoral programs collaborating in the Mid-Atlantic Center for Mathematics Learning and Teaching are committed to the proposition that mathematics education should focus on helping students develop deep, broad, and connected understanding of mathematics. Understanding is the foundation for flexible and efficient mathematical skills, for ability to solve important problems, and for a disposition to learn new mathematical ideas and techniques whenever the need or opportunity occurs. For this reason, the first Foundations of Mathematics Education course offered by the Center universities focuses on theories and research that describe mathematical thinking and understanding and that help in explaining how such thinking and understanding develop.

Course Goals and Activities: This course aims to develop understanding and dispositions of mathematics education doctoral students in five key areas:

- Knowledge of key questions addressed by theories of mathematics learning and the psychological constructs that are useful in formulating such theories.
- Knowledge of prominent classic and contemporary theories of learning that are relevant to study of mathematics learning.
- Knowledge of applications and limitations of theories and research about understanding and learning in core areas of the mathematics curriculum.
- Understanding how theories emerge and are tested by research and application.
- Valuing the search for understanding of student thinking and learning, valuing and questioning evidence about validity of major theories, and becoming reflective about one's own learning.

The course will engage students and faculty in analysis of personal and community beliefs about mathematical thinking and learning, reading of relevant literature, local and videoconference discussions, study of mathematical thinking by students at various levels of schooling, analysis of student work, formulation of evidence-based theories about mathematical thinking and learning, formulation of research questions and strategies, personal reflection on mathematics learning.

Topics: The central topics of the course will be developed in 15 class meetings—possibly including one all-day meeting where students and faculty from all three universities meet face-to-face and14 local site only classes. The sessions and their topics are tentatively planned to unfold as follows:

Sept. 12	Course overview, learning theories; learning goals,
Sept. 19	Studies that exemplify use of different learning theories
Sept. 26	Writings reflective of different theories of learning within mathematics education
Oct. 3	What do we know about how children learn early number facts?
Oct. 10	What do we know about how children learn other early number ideas?

Saturday, October 15, 2005 Tri-campus meeting: Interviewing; (Mis)conceptions

Oct. 17 What seems to make the learning of rational number so difficult?

Oct. 24 What seems to make the learning of rational number so difficult?

Oct. 31 What do we know about how Algebra and Advanced Mathematics learned?

Nov. 7 What do we know about how Algebra and Advanced Mathematics learned?

Nov. 14 Prior Knowledge: Supporter or inhibitor learning?

Nov. 21 What is the relationship between use of representations and understanding?

Nov. 28 What is the relationship between use of representations and understanding?

Dec. 5 What role does community play in learning?

Dec. 12 What role does community play in learning?

Dec.19 Final Exam Due

Course Requirements and Evaluation:

There will be reading and thinking (knowledge worrying) assignments in preparation for each class meeting. Class meetings will also involve small and large group interactions. There will also be one or two brief written assignments at the beginning of the semester. Your oral and written contributions are vital, and 1/8 of your final grade will be based on this work.

Because the course focuses on analysis of student mathematical thinking and learning, there will be a major course project in which you will design and use an interview protocol to study the thinking of one or several students. The design and report of this study will constitute 1/4 of your final grade.

Because the course focuses on research about learning in mathematics, there will be a project, that you may connect to the interview project, requiring you to read and then write a research brief on the learning of a specific mathematics topic of your selection. This research brief will constitute 1/4 of your final grade.

Because we consider knowledge of theories and research about mathematical thinking and learning to be essential in the repertoire of mathematics education doctoral students, there will be a take-home comprehensive written final examination asking you to integrate and apply ideas and information of the course. This exam will constitute 1/4 of your final grade.

As you are all preparing for careers that are likely to involve you as an instructor, one assignment will involve preparation and leading of a discussion/activity related to one or two course readings. This activity will make up 1/8 of your grade.

While each student will be expected to respond to the assignments individually, we encourage you to discuss readings, research tasks, and other assignments with other students and with the faculty.

The University has a nationally recognized Honor Code, administered by the Student Honor Council. The Student Honor Council proposed and the University Senate approved an Honor Pledge. The University of Maryland Honor Pledge reads:

> *"I pledge on my honor that I have not given or received any unauthorized assistance on this assignment/examination."*

Unless you are specifically advised to the contrary, the Pledge statement should be handwritten and signed on the front cover of all papers, projects, or other academic assignments submitted for evaluation in this course. Students who fail to write and sign the Pledge will be asked to confer with the instructor.

Plagiarism is, unfortunately a common form of dishonesty. If you have any questions about the definition or seriousness of this, please read http://www.jpo.umd.edu/SHC/students.html

If you have a documented disability and wish to discuss academic accommodations, please see the instructor as soon as possible.

Goals and Readings for Main Course Topics:

The topics of this course will be presented in two main parts. The goals of these segments and tentative recommended readings are as follows:

1. Perspectives on Mathematics Learning

The goals of these sessions are: (1) To identify the major perspectives on mathematical thinking and learning, (2) To analyze the influence of these theories on practice of school mathematics; (3) To identify key constructs for analysis and theory about mathematics learning and development.

Local Session 1: Sept 12:

Greeno, J.G., Collins, A. M., & Resnick, L. (1996). Cognition and learning. In D.C. Berliner & R. C. Calfee (Eds.), *Handbook of Educational Psychology* (**pp. 15-26**). New York: Macmillan.

Local Session 2: Sept 19

Greeno, J.G., Collins, A. M., & Resnick, L. (1996). Cognition and learning. In D.C. Berliner & R. C. Calfee (Eds.), *Handbook of Educational Psychology* (**pp. 26-41**). New York: Macmillan.

Saxe, G. B. (1988). Candy selling and math learning. *Educational Researcher*, *17*, 14-21.

Mulligan, J., & Mitchelmoore, M. (1997). Young children's intuitive models of multiplication and division. *Journal for Research in Mathematics Education*, *28*, 309-330.

Repp, A.C. (1930). Mixed vs. isolated drill. In *Report of the Society's Committee on Arithmetic: Research in Arithmetic Volume: 29 Issue: 2* (29th NSSE Yearbook, pp. 535- 549).

or

Cox (1975) Systematic errors in the four vertical algorithms in normal and handicapped populations. *Journal for Research in Mathematics Education, 6,* 202-221.

Local Session 3: September 26

**Brownell, W. (1947). The place of meaning in the teaching of arithmetic, *Elementary School Journal*, *47*, 256-265.

And one of

Bruner, J. (1960). *The process of education* (Chapters 2-4, pp 17-68). New York: Vantage.

Dienes (1960). A theory of mathematics-learning. In *Building up mathematics* (Chapter 2, pp. 13-30) London: Hutchinson Educational.

Zazkis, R., & Campbell. S. (1996). Divisibility and multiplicative structure of natural numbers: Preservice teachers understanding. *Journal for Research in Mathematics Education*, *77*,540-563

Davis, R., Maher, C.A., & Noddings, N.(Eds.). (1990). *Constructivist views on the teaching and learning of Mathematics* (JRME Monograph #4), (Part I: Constructivism: Promise and problems, pp. 1-47) Reston, VA: National Council of Teachers of Mathematics.

Ginsbrug, H., & Opper, S. Chapter I (Biography and basic ideas) and Chapter VI (Genetic epistemology and the implications of Piaget's findings for education) in *Piaget's theory of intellectual development: An introduction.* Englewood Cliffs, NJ: Prentice Hall.

Saxe, G. (1991). Part I: Culture and Cognition: A method of study. In Saxe, G. *Culture and cognitive development* (pp.1-18). Hillsdale, NJ: Erlbaum.

Skemp, R. *The psychology of learning mathematics* (Chapters 1-4, pp. 1-45) Hillsdale, NJ: Erlbaum.

Thorndike, E. L. (1929). Chapters V, VI, and VII. In *The psychology of arithmetic* (pp. 102-156). New York: MacMillan

2. Research on Understanding/Learning various mathematical topics

In this second part of the course we will turn our attention to looking at selected research summaries and studies about learning mathematics in three topic areas: early number and addition/subtraction; 2. Rational Number ideas, and 3. Algebra and Advanced Mathematics.

How do Children Learn Early Number Ideas?

The goals of the next two sessions are: (1) To identify the differences in children's mathematical thinking before entering school and after receiving conventional school instruction; (2) To develop possible explanations for these differences by considering previously examined theoretical constructs; an (3) to examine research studies on young students' understandings of early arithmetic concepts and computation.

Local Session 4: October 3

Kilpatrick, J., Swafford, J., & Findell, B. (2001). Chapter 5: The Mathematical knowledge children bring to school and Chapter 6: Developing proficiency with whole numbers. In *Adding it up: Helping children learn mathematics*(pp. 157-230). Washington, DC: National Academy Press.

Carpenter, T. P., Hiebert, J., & Moser, J.M. (1983). The effect of instruction on children's solutions of addition and subtraction word problems. *Educational Studies in Mathematics, 14*, 55-72.

Carpenter, T. P., Ansell, E., Franke, M. L., Fennema, E., & Weisbeck, L. (1993). Models of problem solving: A study of kindergarten children's problem-solving processes. *Journal for Research in Mathematics Education, 24*, 428-441.

Local Session 5: October 10: Interview Question Due

Cobb, P., & Wheatley, G. (1988). Children's initial understanding of ten. *Focus on Learning Problems in Mathematics, 10*(3), 1-28.

Murray, H., Olivier, A., & Human, P. (1992). The development of young students' division strategies. In *Proceedings of the Sixteenth PME Conference,* Vol. II (pp, 2-152 - 2-159). Durham: University of New Hampshire.

Greer, B. (1994). Extending the meaning of multiplication and division. In G. Harel & J. Confrey (Eds.), The development of multiplicative reasoning (pp. 62-85). Albany, NY: State University of New York Press.

Steffe, L. (1994). Children's multiplying schemes. In G. Harel & J. Confrey (Eds.), *The development of multiplicative reasoning* (pp. 3-40). Albany, NY: State University of New York Press.

Tri-campus meeting: Saturday, October 15, 2005 Interviewing; (Mis)conceptions

What makes the learning of rational number so difficult?

The goals for this session are: (1) To recognize the difficulties children have in learning about rational numbers and proportional reasoning and some of the reasons or those difficulties, (2) learn researchers known for their work and or active in this field.

Local Session 6: October 17

Kilpatrick, J., Swafford, J., & Findell, B. (2001). Chapter 7: Developing proficiency with other numbers. In *Adding it up: Helping children learn mathematics* (pp. 231-254). Washington, DC: National Academy Press..

**Erlwanger, S. H. (1973). Benny's Conception of rules and answers in IPI mathematics, *Journal of Children's Mathematical Behavior, 1*, 7-26.

Glaser, R. (1965). Toward a behavioral science base for instructional design. In R. Glaser (Ed.), *Teaching machines and programmed learning, II, data and directions* (pp. 771-809). Washington, DC: National Education Association of the United States.

And one of:

Cramer, K., Post, T., & delMas, R. (2002). Initial fraction learning by fourth –and fifth-grade students: A comparison of the effects of using commercial curricula with the effects of using the Rational Number Project Curriculum, *Journal for Research in Mathematics Education, 33,*111-145.

Post, T., Wachsmuth, I., Lesh,P., & Behr, M.J. (1985). Order and equivalence of rational numbers: A cognitive analysis. *Journal for Research in Mathematics Education. 16*,18-36.

Local Session 7: October 24: Topic Research Brief Due

Heller, P., Post, T., Behr, M., & Lesh, R. (1990). Qualitative and numerical reasoning about fractions and rates by seventh- and eighth-grade students. *Journal for Research in Mathematics Education, 21*(5), 388-402.

Quintero, A. (1983). Conceptual understanding in solving two-step word problems with a ratio. *Journal for Research in Mathematics Education, 14*(2), 102-112

Lamon, S. J. (1995). Ratio and proportion: Elementary didactical phenomenology. In J. S. a. B. Schappelle (Ed.), *Providing a foundation for teaching mathematics in the middle grades* (pp. 167-198). Albany: State University of New York Press.

Ben-Chaim, D., Fey, J. T., Fitzgerald, W., Benedetto C., & Miller, J. (1998). Proportional reasoning among 7th grade students with different curricular experiences. *Educational Studies in Mathematics, 36*, 247-273.

How are Algebra and Advanced Mathematics learned? Examining research on learning of core concepts and skills in algebra and functions strand of school mathematics.

Local Session 8: October 31 : Interview Protocol Due

Chazan, D., & Yerushalmy, M. (2003). On appreciating the cognitive complexity of school algebra: Research on algebra learning and directions of curricular change. In J. Kilpatrick, W. G. Martin, & Schifter, D. (Eds.), *A Research Companion to Principles and Standards for School Mathematics* (pp.123-135). Reston, VA: National Council of Teachers of Mathematics.

Kieran, C. (1981). Concepts associated with the equality symbol. *Educational Studies in Mathematics*, *12*, 317-326.

Wagner, S. (1981). Conservation of equation and function under transformations of variable. *Journal for Research in Mathematics Education*, *12*, 107-118.

Local Session 9: November 7

Sfard, A., & Linchevki, L. (1994). The gains and the pitfalls of reification—the case of algebra. *Educational Studies in Mathematics, 26*,191-228.

Graham, A.T., & Thomas, M. O. (2000). Building a versatile understanding of algebraic variables with a graphic calculator. *Educational Studies in Mathematics, 42*, 265-282.

3. Enduring Questions: We pursue three questions of current and continued interest and look at some of the writing and research surrounding those questions. In doing so we revisit some topics and theories that we have previously read.

Prior Knowledge: Supporter or inhibitor learning?

Local Session 10: November 14

Smith, J, P., diSessa, A., & Roschelle, J. (1993/4). Misconceptions reconceived: A constructivist analysis of knowledge in transition. *The Journal of The Learning Sciences 3,* 115-156.

Mack, N. (1990). Learning fractions with understanding: Building on informal knowledge. *Journal for Research in Mathematics Education, 21*, 16-32.

Fischbein, E., Deri, M., Nello, M., & Marino, M. (1985). The role of implicit models in solving problems in multiplication and division. *Journal for Research in Mathematics Education, 16*(2).

Confrey, J. (1994). Splitting, similarity, and rate of change: A new approach to multiplication and exponential functions. In G. Harel & J. Confrey (Eds.),*The development of multiplicative reasoning in the learning of mathematics* (pp.293-332). Albany, NY: State University of New York Press.

What is the role of representations in understanding?

Local Session 11: November 21

Goldin, G., & Shteingold, N. (2001). Systems of representations and the development of mathematical concepts. In A. Cuoco & F. Curcio (Eds.), *The roles of representation in school mathematics* (2001 Yearbook of the NCTM) (pp 1-23), Reston, VA: National Council of Teacher of Mathematics.

**Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, *12*, 151-169.

Yerushlamy, M., & Shternberg, B. Charting a visual course to the concept of function. In A. Cuoco & F. Curcio. (Eds.), *The roles of representation in school mathematics* (2001 Yearbook of the NCTM) (pp. 251-268). Reston, VA: National Council of Teachers of Mathematics.

Pimm, D. (1987). The mathematics register. In D. Pimm *Speaking mathematically: Communication in the mathematics classrooms* (pp. 75-110). New York: Routledge.

Local Session 12: November 28

Hiebert, J., Wearne, D., & Taber, S. (1991) Fourth graders' gradual construction of decimal fractions during instruction using different physical representations. *Elementary School Journal, 91,*321-341.

Ruthven, K. (1990). The influence of graphic calculator use on translation from graphic to symbolic forms. *Educational Studies in Mathematics*, *21*, 431-450.

Dufour-Janivier, B., Bednarz, N., & Blanger, M. (1987). Pedagogical considerations concerning the problem of representation. In C. Janvier (Ed.) *Problems of representation in the teaching and learning of mathematics* (pp.109-122). Hillsdale, NJ: Lawrence Erlbaum.

What role does community play in learning?

Local Session 13: December 5: Interview Analysis Due

Forman, E. A. (2003). A sociocultural approach to mathematics reform: Speaking, inscribing, and doing mathematics within communities of practice. In J. Kilpatrick, W. G Martin & D. Schifter (Eds.), *A research companion to Principles and Standards for School Mathematics,* pp. 333-353. Reston, VA: National Council of Teachers of Mathematics.

Kozulin, A. (1998). *Psychological tools: A sociocultural approach to education* (Chapter 2: Piaget, Vygotsky, and the Cognitive revolution, pp.34-58). Cambridge, MA: Harvard University Press.

Local Session 14: December 12

Cobb, Wood, & Yackel (1993). Discourse, mathematical thinking and classroom practice. In E. Forman, N. Minickk, & A. Stone (Eds.), *Contexts for learning: Sociocultural dynamics in children's development* (pp. 91-119). New York: Oxford University Press.

O'Connor, M.C. (2001). "Can any fraction be turned into a decimal?" A case of a mathematical group discussion. *Educational Studies in Mathematics*, *46*, 143-185.

Yackel, E. Children's talk in inquiry mathematics classrooms. In P. Cobb & H. Bauersfeld (Eds.). *The emergence of mathematical meaning: Interaction in the classroom*. Mahwah, NJ: Erlbaum.

December 19, 2005 Final Examination Due

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